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An approximate variance estimator for index of population trend developed with delta-method and its application

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Abstract: On the basis of Watt's mathematical model of the index of population trend (I), an approximate variance estimator of I was developed with delta-method in this paper. Taking the natural population life tables of rice leaf roller *Cnaphalocrocis medinalis* (Lepidoptera: Pyralidae) published by Wu *et al*. (1986) as an example, it was applied in evaluating the control effectiveness of biological agent (*Trichogramma japonicum* Ashmead) and chemical insecticide (powder, 1.5% mevinphos + 3% alpha-hexachloro cyclohexane). By Z-test criterion, biologically and statistically sound conclusions were drawn that at the generation level, the suppression effectiveness of trichogramma wasp against rice leaf roller was better than the blank control with $P = 0.0111$ and smaller I value (0.0390 versus 0.1768), and it was also statistically better than the insecticides with $P = 0.0036$ and smaller I value (0.0390 versus 0.3035). The insecticides was not statistically worse than the blank control with greater I value (0.3035 versus 0.1768), and $P = 0.2236$. Although these conclusions are close to that drawn by Wu *et al*. (1986) only based on the index of population trend point estimates, they are supported by an accuracy measurement P-value.
Key words: Approximate variance estimator; index of population trend; life table; rice leaf roller (*Cnaphalocrocis medinalis*); delta-method

Since Morris and Miller (1954) introduced life table methodology into insect natural population dynamics analysis, this method has been widely applied in evaluating the effectiveness of physical or biological factors on insect population fluctuation, and also in determining the management effectiveness of chemical or biological agents in agricultural pest management practice. Watt (1961, 1963) developed mathematical models computing the index of population trend (I), which provided an integrated index containing all regulation effect information of environmental factors on the whole generation of the target population. Harcout (1963, 1969) discussed life table construction tactics, its application scenario, and future applications in details. Pang *et al*. (1981) discussed field experiment design, data collection, and survival rate estimation tactics for constructing a type of natural population life table of rice leaf roller (*Cnaphalocrocis medinalis*), and its application in determining the effectiveness of environmental factors. By the rice leaf roller natural population life tables, Pang *et al*. (1984) used index of population trend as the analytical tool evaluating the management effectiveness of chemical and biological agents in its IPM system at generation level, and concluded that at generation level biological agents were not only environmental friendly but also more effective and efficient than chemical insecticides. Bellows *et al*. (1992) and Carey (2001) reviewed the recent development in life table methodology focusing on experimental design, data collection, and mortality estimation, and its application in analyzing insect population dynamics, estimating the effectiveness of environmental factors, such as natural enemies and host plant traits, and determining control effectiveness of biological and physical factors in pest management. Unfortunately, up to date there is still no publication available focusing on the variance estimation of index of population trend (I). By Watt's mathematical model and delta-method, the approximate variance estimation of the index of population trend and its application are the topics concerned in the following sections.

1 AN APPROXIMATE VARIANCE ESTIMATOR FOR INDEX OF POPULATION TREND (I)

1.1 Mathematical model of index of population trend (I)

By the mathematical model developed by Watt

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(1961, 1963), Pang *et al*. (1984) applied the following equation computing the index of population trend (I) for natural population life table of rice leaf roller,

$$I = S_e S_l \Lambda S_n S_p S_f S_r N_e \quad \cdots\cdots (1)$$

Where S_e is the survival rate at egg stage, $S_l \cdots, S_n$ is the survival rate at first, …, last larva stage respectively, S_p is the survival rate at pupa stage, S_f is the female moth rate, S_r is the proportion of female moth achieving standard egg number calculated with the actual mean egg number divided by theoretical standard egg number, N_e is the standard egg number per female moth.

1.2 Delta-method

Delta-method based on Taylor expansion is often applied to estimate the approximate variance and the corresponding confidence intervals for a random variable for which there is not explicitly mathematical formula available to estimate its variance (Sorensen and Gianola, 2002). Let x, y be two random variables with the following function relation,

$$y = f(x)$$

And assume that this function is differentiable at least twice. Let x_0 be the expected value of x. If the value of $f(x)$ is known at the point of x_0, denoted by $f(x_0)$, the value of y at x (which is pretty close to its expectant) can be approximately formulated by Taylor expansion as follows.

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)^2 (x - x_0)^2$$

Remove the constant term $f(x_0)$ to the left of the equal sign of the previous equation, there is

$$f(x) - f(x_0) \approx f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)^2 (x - x_0)^2$$

If the variance of random variable x is known, the variance of random variable y can be approximately estimated by the following mathematical formula through only retaining the second order term,

$$\mathrm{var}(y) = \mathrm{var}(f(x) - f(x_0)) \approx (f'(x_0))^2 \mathrm{var}(x) \cdots (2)$$

Or if the variance of random variable y is known, the variance of random variable x can be approximately estimated by

$$\mathrm{var}(x) \approx (f'(x_0))^{-2} \mathrm{var}(y) \quad \cdots\cdots (3)$$

1.3 An approximate variance estimator for index of population trend (I)

Transforming equation (1) of the index of population trend (I) with logarithm, there is

$$\log(I) = \log(S_e) + \log(S_1) + \Lambda + \log(S_n) + \log(S_p) + \log(S_f) + \log(S_r) + \log(N_e) \quad \cdots\cdots (4)$$

By delta-method, the variance of log-transformed survival rate is easily calculated,

$$\mathrm{var}(\log(S_i)) \approx \mathrm{var}(S_i)/S_i^2 \quad \cdots\cdots (5)$$

Since the survival rate of released cohort of rice leaf roller is related to binomial distribution, the variance of the survival rate (S_i) is computed with the following formula,

$$\mathrm{var}(S_i) = \frac{S_i * (1 - S_i)}{n_i}$$

Here n_i is the number of individuals released at ith life stage. Actually, the true survival rate is unknown. The maximum likelihood estimate of survival rate ($\hat{S}_i$) is substituted into previous mathematical formula, and the maximum likelihood variance estimate of the survival rate is gotten. With similar procedure as survival rate, the variance of female moth rate is estimated.

For the reproductive component at adult stage, denote the mean egg number per female moth and the corresponding variance as $\bar{x}$, and $\mathrm{var}(\bar{x})$, which are obtained from cohort experiment. The proportion of female moth achieving standard egg number is $S_r = \frac{\bar{x}}{N_e}$. The corresponding variance is $\mathrm{var}(S_r) = \frac{\mathrm{var}(\bar{x})}{N_e^2}$. The variance of logarithm transformed female moth proportion achieving standard egg number is

$$\mathrm{var}(\log(S_r)) = \left(\frac{\mathrm{N_e}}{\bar{\mathrm{x}}}\right)^2 \mathrm{var}(\mathrm{S_r}) = \frac{\mathrm{var}(\bar{\mathrm{x}})}{\bar{\mathrm{x}}^2}$$

By the field experiment design and data collection tactics with release and recapture method, the independence among sequential life stages is assumed for simplicity. Thus,

$$\mathrm{var}(\log(I)) \approx \sum_{j=1}^{m} \sum_{i=1}^{nj} \mathrm{var}(\log(S_{ji})) \quad \cdots\cdots (6)$$

Where m is the total life stages, n_j is components in jth life stage. The standard egg number for a female moth (N_e), a constant, vanishes in the variance estimation model. By the variance estimates for $\log(I)$, the variance of the index of population trend (I) is

$$\mathrm{var}(I) \approx \exp(\log(I))^2 \mathrm{var}(\log(I)) = I^2 \mathrm{var}(\log(I)) \quad \cdots\cdots (7)$$

Since the true value of index of population trend is also unknown, after substituting its maximum likelihood estimate ($\hat{I}$) into this equation, its maximum likelihood variance estimator is,

$$\mathrm{var}(\hat{I}) \approx \exp(\log(\hat{I}))^2 \mathrm{var}(\log(\hat{I})) = \hat{I}^2 \mathrm{var}(\log(\hat{I})) \quad \cdots\cdots (8)$$

1.4 Confidence intervals for index of population trend (I) and Z-test criterion

By central limit theorem, when the sample size is large enough, the following statistics approximately

tends to standard normal distribution,

$$\frac{\hat{I} - I}{\sqrt{\mathrm{var}(\hat{I})}} \xrightarrow{n \to \infty} N(0,1)$$

Where n is the sample size, I the true value of index of population trend, and $N(0,1)$ the standard normal distribution. By this law, the $1-\alpha$ confidence intervals of the index of population trend (I) is estimated as

$$(\hat{I} - Z_\alpha \times \sqrt{\mathrm{var}(\hat{I})}, \hat{I} + Z_\alpha \times \sqrt{\mathrm{var}(\hat{I})}) \quad \cdots\cdots (9)$$

Here $Z_{\frac{\alpha}{2}}$ is the critical value from standard normal distribution at $1-\frac{\alpha}{2}$.

Depending on the null hypothesis that there is no difference of index of population trend between two treatments $(I_i - I_j = 0)$, and the independence between treatments guaranteed by the experimental design, the following Z-test criterion is applied in treatment comparison,

$$Z = \frac{(\hat{I}_i - \hat{I}_j)}{\sqrt{\mathrm{var}(\hat{I}_i) + \mathrm{var}(\hat{I}_j)}} \quad \cdots\cdots (10)$$

By this Z-value, for two tail test, the corresponding P-value is easily obtained.

2 APPLICATION OF THE APPROXIMATE VARIANCE ESTIMATOR

2.1 Life stage specific life table of rice leaf roller and the hypothesis test

Based on the life tables of rice leaf roller from three field treatments (blank control plot, biological control plot released with *Trichogramma japonicum* Ashmead, and insecticide control plot dusted with 1.5% mevinphos + 3% alpha-hexachloro cyclohexane powder) published by Wu *et al*. (1986), their corresponding life stage specific life table, the maximum likelihood estimate of the index of population trend (I) for each treatment, and its approximate variance estimate are listed in Table 1. In insect population management experiment, the null hypothesis (H_o) assumes that the control agent does not change survival and reproduction patterns of the target host population or the control agent does not change the index of population trend (I), which is tested with life table methodology. The alternative hypothesis (H_a) is that the control agent significantly changes the survival and reproduction patterns of the target host population or it significantly changes the index of population trend with type I error probability $\alpha = 0.2$, 0.1, 0.05, or 0.01 respectively. By Z-test criterion, the index of population trend (I) in each treatment and their 80%, 90%, 95%, 99% confidence intervals are plotted in Fig. 1. By the confidence intervals of I, when the type I error $\alpha = 0.05$, the biological control plot was statistically better than the blank control and the insecticide treated plot. When the type I error $\alpha = 0.01$, there was no difference among all the three treatments. By the comparison between treatments with Z-test criterion, the suppression effect of releasing wasp was statistically better than blank control with smaller I-value (0.0390 versus 0.1768) and P-value (0.0111) < 0.05, and also statistically better the chemical insecticide with smaller I-value (0.0390 versus 0.3035) and P-value (0.0036) < 0.01. The chemical insecticide was not statistically worse than blank control with greater I-value (0.3035), and P-value (0.2236) > 0.05. Between these to procedures, the confidence interval comparison is much more conservative. Although such results are close to that drawn only by the point estimates of index of population trend, these conclusions are supported by an accuracy measurement P-value.

Table 1 Natural population life table of rice leaf roller

Life Stage or Parameter		Survival rate		
		Blank control	*Trichogramma* wasp	Insecticides
Egg		0.4130	0.1850	0.4160
Larva	1st instar	0.4400	0.3400	0.2000
	2nd instar	0.5000	0.3100	0.6200
	3rd instar	0.3000	0.3300	0.5400
	4th instar	0.7000	0.5600	0.7600
	5th instar	0.5500	0.6000	0.6500
Pupa		0.4100	0.3800	0.5400
Adult	S_f	0.5000	0.5000	0.5000
	S_r	0.4100	0.4100	0.4100
	Ne	200	200	200
	$\hat{I}$	0.1768	0.0390	0.3035
Variance of $\hat{I}$		0.0028	0.0002	0.0082

3 CONCLUSIONS

In data analysis, the point estimates for population parameters are useful to show population properties, such as mean, median, quartiles, *etc*. When such estimates are applied to compare properties of different populations, such as insect on different host plants or on different resistance level cultivars, in different geographical areas, and with different genetics traits, it is impossible to draw statistically dependable conclusions only by point estimates, although such a comparison does provide certain information. Thus, how to estimate the variance corresponding to the point estimate is a pragmatic problem related to management agent selection in pest IPM system. Unfortunately, for some statistics, like the index of population trend (I), it is very difficult to find an explicit mathematical model to calculate its variance, up to date there is still no such estimator available. Practically, it is desirable

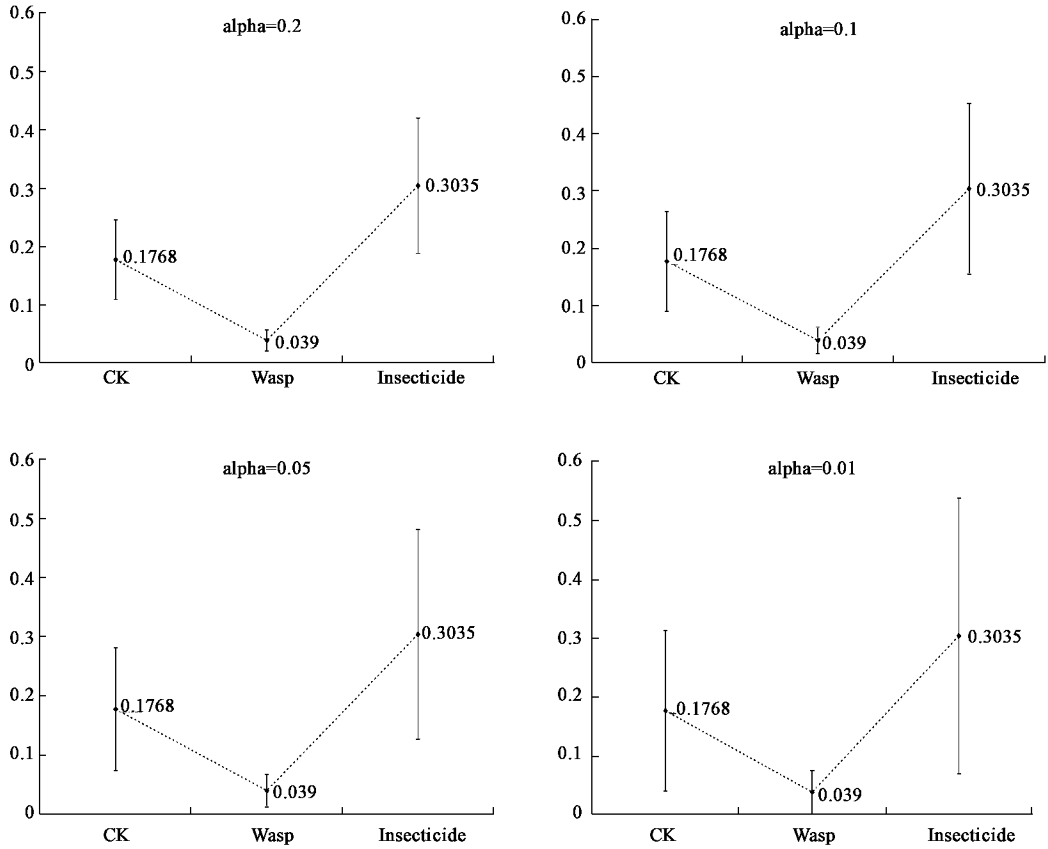

Fig. 1 Index of population trend and its $1-\alpha$ confidence intervals in control, wasp, and insecticide treated plots

using approximate estimator achieving variance estimates for population point estimates. By the analysis and discussion in foregoing sections, the following conclusions are drawn:

(1) After ignoring the correlations between the sequential life stages of the target insect population by the field experiment design, sampling schedule, and data collection tactics, an approximate variance estimator for the index of population trend is obtained by the log-transformed Watt's mathematical model and delta-method.

(2) By the central limit law, when the sample size is large enough, the estimates of the index of population trend (I) tends to confirm with $N(I, \sigma^2)$, where σ^2 is variance estimated by the approximate variance estimator. By this law, $(1-\alpha)$ confidence intervals can be estimated for each point estimate of the index of population trend, and the Z-test criterion can be applied into the evaluation of the pest management agents.

(3) By Z-test criterion, the biological control plot was statistically superior to the blank control (P = 0.0111) and the chemical treated plot ($P = 0.0036$), but the chemical treated plot did not differ significantly from control with P = 0.2236. Although such conclusions are close to that drawn by Wu *et al*. (1986) only based on the point estimates of index of population trend, they are supported by an accuracy measurement P-value and statistically dependable.

This approximate variance estimator for index of population trend (I) is based on the following assumptions: (a) the distribution of the index of population trend (I) tends to normal when sample size is large enough, and (b) the correlation between survival rates of the sequential life stages is negligible. Actually, it is very difficult to answer whether the normality of the estimates is achieved and the correlations between the survival rates of sequential life stages are negligible. By biological relations within the interaction system composed of rice plant, rice leaf roller, its natural enemy, and its food competitors, more or less there exists direct or indirect correlation between survival rates of the sequential life stages. Thus, searching for rational variance estimating techniques for the index of population trend (I) based on life table data is still a question worthy of future investigation.

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种群趋势指数的方差近似估计式及其应用

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摘要:根据吴惠龙等(1986)发表的稻纵卷叶螟 *Cnaphalocrocis medinalis* 的生命表及 Watt 种群趋势指数数学模型,在 delta-方法的基础上,本文提出了一种种群趋势指数方差的近似估计法,并应用于评价稻螟赤眼蜂 *Trichogramma japonicum* Ashmead 及化学杀虫剂(甲六粉)对稻纵卷叶螟的控制作用。根据中心极限定律,当样本足够大时,种群趋势指数近似于正态分布。根据近似方差估计量以及 Z-检验方法,取得以下结论:放蜂区明显优于对照区,因其种群趋势指数取值较小,相应的 P 值为 0.0111,该区也明显优于化学杀虫剂处理区,相应的 P 值为 0.0036。杀虫剂处理区差于对照区,因其种群趋势指数值较大(0.3035),但这种差异并未达到显著水平,P 值为 0.2236。虽然本文的结论近似于吴惠龙等(1986)单纯依据种群趋势指数所得到的结果,但本文的结论更具统计意义,更加可靠。

关键词:近似方差估计式;种群趋势指数;生命表;稻纵卷叶螟;delta-方法
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